

$$3) y + \ln xy = 1$$

$$\frac{dy}{dx} + \left(\frac{1}{xy} \left(x \frac{dy}{dx} + (1y) \right) \right) = 0$$

$$\frac{dy}{dx} = \left(\frac{x \frac{dy}{dx}}{xy} + \frac{y}{xy} \right)$$

$$\frac{dy}{dx} = \left(\frac{1}{xy} \left(x \frac{dy}{dx} + (y) \right) \right)$$

$$\frac{dy}{dx} + \frac{1}{y} \frac{dy}{dx} = -\frac{1}{x}$$

$$\frac{dy}{dx} \left(1 + \frac{1}{y} \right) = -\frac{1}{x}$$

$$xy \left(\frac{dy}{dx} + \left(\frac{x \frac{dy}{dx}}{xy} + (y) \right) \right) = xy \left(\frac{dy}{dx} \right) = - \left(x \frac{dy}{dx} + (y) \right)$$

$$xy \left(\frac{dy}{dx} \right) = - \left(\frac{x \frac{dy}{dx}}{\frac{dy}{dx}} + \frac{(y)}{\frac{dy}{dx}} \right) = xy = - \left(x \frac{dy}{dx} + (y) \right)$$

$$xy = -x - \left(\frac{y}{\frac{dy}{dx}} \right) \frac{dy}{dx} \left(x + xy \right) = \left(\frac{y}{\frac{dy}{dx}} \right) \frac{dy}{dx}$$

$$\left(\frac{dy}{dx} \right) \left(x + xy \right) = -x$$

$$\frac{dy}{dx} = \frac{-y}{x(1+y)}$$

$$\ln(x \tan(y)) = \ln\left(\frac{1}{x \tan(y)}\right) \cdot \left((1 \tan(y)) + (x \sec^2(y)) \left(\frac{dy}{dx}\right)\right) = \frac{dy}{dx}$$

$$\frac{\tan(y) + (x) \sec^2(y) \left(\frac{dy}{dx}\right)}{(x \tan(y))} = \frac{dy}{dx}$$

$$\frac{\left(\frac{\cos(y)}{1}\right) \frac{\sin(y)}{\cos(y)} + \frac{x}{\cos(y)} \left(\frac{dy}{dx}\right) \left(\frac{1}{\cos(y)}\right)}{x \frac{\sin(y)}{\cos(y)} \left(\frac{\cos(y)}{1}\right)} = \frac{dy}{dx}$$

$$\frac{x \sin(y) \sin(y) + \frac{x}{\cos(y)} \left(\frac{dy}{dx}\right)}{x \sin(y)} = \frac{dy}{dx} (x \sin(y))$$

$$\sin(y) + \frac{x}{\cos(y)} \left(\frac{dy}{dx}\right) = \frac{dy}{dx} (x \sin(y)) - \frac{x}{\cos(y)} \left(\frac{dy}{dx}\right)$$

$$\sin(y) = \frac{dy}{dx} \left(x \sin(y) - \frac{x}{\cos(y)} \right)$$

$$\frac{\sin(y) - \frac{x}{\cos(y)}}{x \sin(y) - \frac{x}{\cos(y)}} = \frac{dy}{dx}$$

$$\frac{\sin(y) - \frac{x}{\cos(y)}}{x \sin(y) - \frac{x}{\cos(y)}} = \frac{dy}{dx}$$

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$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{(e^x(1) - e^{-x}(-1))(e^x + e^{-x}) - (e^x - e^{-x})(e^x + e^{-x})(-1)}{(e^x + e^{-x})^2}$$

$$(e^x + e^{-x})(e^x + e^{-x}) = e^{2x} + e^0 + e^0 + e^{-2x}$$

$$(e^x - e^{-x})(e^x - e^{-x}) = e^{2x} - e^0 - e^0 + e^{-2x}$$

$$(e^{2x} + e^0 + e^0 + e^{-2x}) - (e^{2x} - e^0 - e^0 + e^{-2x})$$

$$\frac{(e^{2x} + 1 + 1 + e^{-2x}) - (e^{2x} - 1 - 1 + e^{-2x})}{(e^x + e^{-x})^2}$$

$$\frac{(e^{2x} + 1 + 1 + e^{-2x}) - (e^{2x} - 1 - 1 + e^{-2x})}{(e^x + e^{-x})^2}$$

$$\frac{(e^{2x} + 1 + 1 + e^{-2x}) - (e^{2x} - 1 - 1 + e^{-2x})}{(e^x + e^{-x})^2}$$

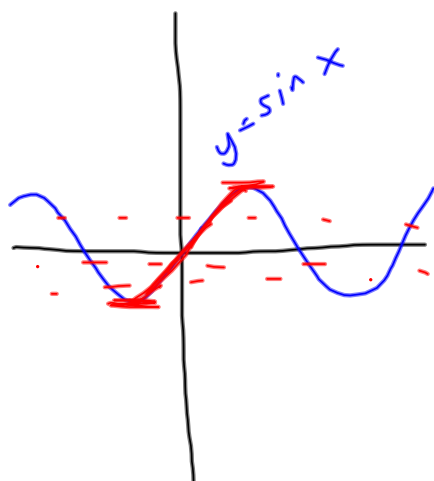
$$= \frac{4}{(e^x + e^{-x})^2}$$

$$\begin{aligned} \frac{d}{dx} \left(\ln \left(\frac{x^2 \sin x}{\sqrt{1+x}} \right) \right) &= \frac{\sqrt{1+x}}{x^2 \sin x} \left(\frac{d}{dx} \frac{x^2 \sin x}{\sqrt{1+x}} \right) \\ &= \frac{\sqrt{1+x}}{x^2 \sin x} \left(\frac{[2x \sin x + x^2 \cos x] \sqrt{1+x} - x^2 \sin x \left[\frac{1}{2\sqrt{1+x}} \right]}{(\sqrt{1+x})^2} \right) \\ &= \frac{1}{x^2 \sin x \sqrt{1+x}} \left(2x \sin x \sqrt{1+x} + x^2 \cos x \sqrt{1+x} - \frac{x^2 \sin x}{2\sqrt{1+x}} \right) \\ &= \frac{x}{x^2 \sin x} \left(\frac{2 \sin x \sqrt{1+x}}{\sqrt{1+x}} + \frac{x \cos x \sqrt{1+x}}{\sqrt{1+x}} - \frac{x \sin x}{2\sqrt{1+x} \sqrt{1+x}} \right) \\ &= \frac{1}{x \sin x} \left(2 \sin x + x \cos x - \frac{x \sin x}{2(1+x)} \right) \end{aligned}$$

$$\frac{1}{\left(\frac{1}{x}\right)} = x$$

$$\frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}$$

4.4 Inverse Trig Functions



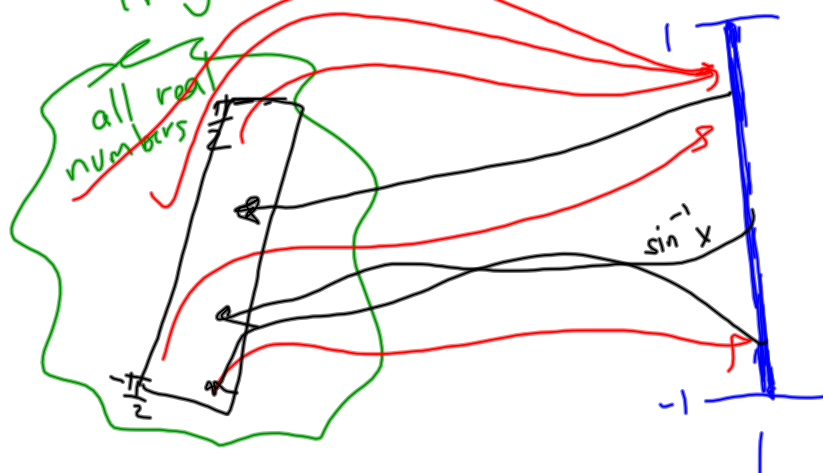
to "make" $\sin x$ invertible,
consider the restricted
domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

call that fn $y = \text{Sin}(x)$

Its inverse $\left[\sin^{-1}(x) \text{ or } \arcsin(x)\right]$, then is

"the angle whose $\text{Sin} = \text{a number}$ "

A "mapping" version of this



Inverse Trig Functions

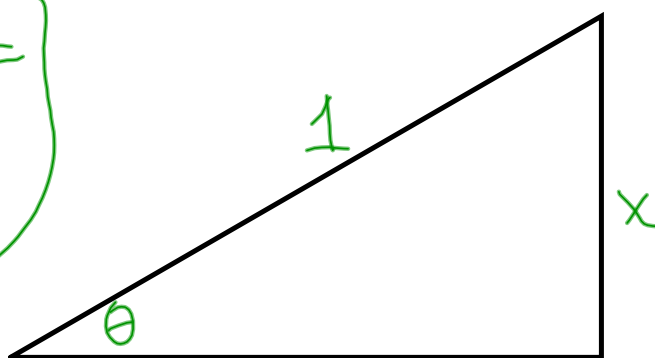
A technique

Can we determine $\cos(\sin^{-1}(x))$?

What is $\sin^{-1}(x)$?

Yes, $\sin^{-1}(x)$ is the angle whose sine is x .

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$



$$\sqrt{1-x^2}$$

$$\text{So } \cos \theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$